Optimal Short-Range Trajectories for Helicopters

G.L. Slater*
University of Cincinnati, Cincinnati, Ohio
and
H. Erzberger†
NASA Ames Research Center, Moffett Field, California

An optimal flight path algorithm using a simplified altitude state model and an a priori climb-cruise-descent flight profile has been developed and applied to determine minimum fuel and cost trajectories for a helicopter flying a fixed-range trajectory. The performance model is based on standard flight manual data and is such that on-line trajectory optimization is feasible with a relatively small computer. The results show that the optimal flight path and cruise altitude can represent a 10% fuel savings on a minimum fuel trajectory. The optimal trajectories show considerable variability due to helicopter weight, ambient winds, and the relative cost tradeoff between time and fuel. In general, "reasonable" variations from the optimal velocities and cruise altitudes do not significantly degrade the cost.

I. Introduction

THE unique hover- and low-speed capabilities of the helicopter have made this vehicle an important mode of transportation for many applications. General usage of helicopters for commercial application has been limited, however, by the relatively high cost of fuel and flight time for this vehicle, in comparison to those of conventional fixed-wing aircraft. While flight path optimization seems particularly attractive as a means of minimizing these costs, two factors have tended to limit the development of specified operational procedures to allow the pilot to fly optimized flight paths. These are: 1) The performance characteristics of the helicopter are quite complex and exhibit wide variation with weight and altitude. These changes cause consequent major differences in the character of the optimal paths. 2) Because the helicopter is a relatively low-speed vehicle, atmospheric winds also play a significant role in determining the shape of the optimal trajectories. Unless accurate knowledge of winds is available, a computed optimal trajectory may be significantly in error.

One of the first attempts at helicopter flight path optimization was done by Schmitz,[1] who investigated the takeoff problem for a heavily loaded helicopter using a variational approach and later tested a "suboptimal" implementation of this control policy.[2] The work of Olsen[3] was directed at on-board optimization of climb and cruise trajecories but utilized only a classical quasi-steady performance approach.

The purpose of this study is to develop a synthesis procedure to allow on-board generation of "optimal" trajectories for helicopters for arbitrary weight and wind conditions. In particular, trajectories are determined which minimize a cost function chosen as a weighted sum of time and fuel such that time, fuel, or cost can be minimized by appropriate selection of the weighting factors. The analytical procedure is based on the method used by Erzberger[4] for fixed-wing aircraft. The method is specialized here to apply to a performance model representative of a helicopter. Much of the effort in this study has been toward the generation of an appropriate performance model which reflects the variability in the true vehicle, yet is simple enough to allow on-board, real-time computation of trajectories. In this report we have specialized our study to the Sikorsky S-6IN helicopter. Section II of this paper outlines the performance model used in this study. The development of the optimization procedure is shown in Section III. The essential characteristics of the optimal trajectories and differences between these results and the comparable fixed-wing results are emphasized. Finally, the application of the optimization algorithm and representative optimal trajectories for the S-6IN helicopter are shown in Section IV.

II. Development of a Helicopter Performance Model

In the development of a performance model for the helicopter, two goals influence our approach. The first is the recognition that the primary aim of this effort is to develop an on-line procedure such that operational helicopters with only moderate onboard comptuational capability can utilize this model to perform on-line trajectory optimization. The second is awareness that the model must be accurate enough over the entire operating envelope of the vehicle so that "optimal" trajectories computed using this model will not deviate significantly from the "true" optimal policy. To implement the flight path optimization algorithm requires three performance quantities. These are:

1) Power required for level flight (cruise power).
2) Rate of climb and rate of descent data.
3) Engine power available and fuel flow data.

These three items are needed for all velocities and altitudes within the vehicle flight envelope, as well as for the range of weights and engine power settings. Our treatment of each of these items is found in succeeding sections.

Cruise Power Required

The difficulties of analytically modelling nonaxial flow through a rotor and accurately accounting for various important effects such as stall, compressibility, etc., make a strictly analytical approach to this problem unreasonable. Yet without some account of the significant effects on required power caused by these items, the mathematics of the optimization procedure frequently lead to invalid (and possibly irrational) results. The approach taken in this Paper is to use only a simple phenomenological model based on momentum and blade element theories. The flight manual data is then

Submitted Aug. 5, 1982; revision received July 29, 1983.

*Professor, Department of Aerospace Engineering and Applied Mechanics. Member AIAA.
†Research Scientist. Member AIAA.

fitted to this simple model using variable coefficients which are functions of vehicle weight and altitude.

We consider the power required for level cruise to be of the form

$$P_R = c_I P_I + c_2 P_O + c_3 P_P \tag{1}$$

The terms P_I, P_O, P_P are analutical models of the induced, parasitic, and rotor profile powers, respectively (see a text such as Bramwell[5] or Gessow and Myers[6]).

By assuming a linear variation of c_I, c_2, and c_3 with altitude for each weight, the coefficients in Eq. (1) are obtained from a least squares fit of the flight manual data.[7] In view of the simplistic assumptions, it is perhaps surprising that Eq. (1) gives an excellent fit to all the flight data with the exception of the high-altitude, high-weight conditions (see Figs. 1 and 2). In this region, large rotor power losses cause the simple power model to be significantly in error. Since, as shown in the sequel, the optimal trajectories generally do not enter this region, this deficiency in the model turns out to be not especially serious.

Climb-Descent Calculations

Unlike for the fixed-wing aircraft, the helicopter level flight cruise power curves cannot be directly applied to computation of the climb performance. The reason for this is that in contrast to the fixed wing aircraft, the helicopter in a climb experiences very little fuselage rotation. Consequently, a vertical velocity component significantly affects the inflow into the rotor, hence the force and power characteristics may change significantly from level flight.

Using a technique suggested by Keys[8] the helicopter climb is simplified by introducing an empirical climb (or descent) factor and writing the climb rate $\dot{h}$ as

$$\dot{h} = k_{(\ )} (P_A - P_R)/W \tag{2}$$

where ( ) indicates either climb or descent, P_A is the rotor shaft power available, P_R is the level flight cruise power required, and W is the vehicle weight. For the climb factor Keys reports typical values as $k_{CLB} \simeq 0.75\text{-}0.85$. The corresponding descent factor $k_{DST} \simeq 1$.

The climb factor for the S-61N is estimated by comparing the flight manual maximum rate of climb data to the computed maximum rate of climb data using Eq. (2). On the basis of this data $k_{CLB} = 0.75$ and $k_{DST} = 1$ are used throughout this paper.

Propulsive Model

The S-61N is powered by dual GE CT58-140 turbine engines. Power and fuel flow characteristics of the two engines are modelled quite well by the linear relations

$$(P)_{\max} = 1250 - h/40 \tag{3}$$

and

$$W_f = 400\delta\sqrt{\theta} + 0.5P \tag{4}$$

where the power P is in horsepower (hp), fuel flow is in lb/h, and altitude is in feet (ft). δ and θ are the pressure ratio and temperature ratio, respectively.

For conventional quasi-steady performance and also for the variational analysis in this paper, an important parameter on fuel optimal trajectories is the cruise cost parameter defined as

$$\min_V [W_f/V]$$

The calculated cruise cost parameter is shown in Fig. 3. The symbols shown in Fig. 3 indicate the actual cruise cost parameter as calculated for each individual weight and altitude. These indicate that the performance model used represents the flight data quite accurately, except for heavy weights at high altitudes.

III. Formulation of Optimal Trajectory Problem

Mathematics of Optimization

Using the helicopter performance model of the previous section, it is in our interest to examine fixed range trajectories

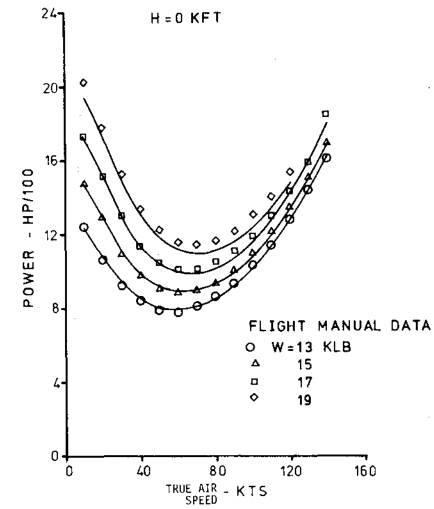

Fig. 1 Performance model fit at sea level.

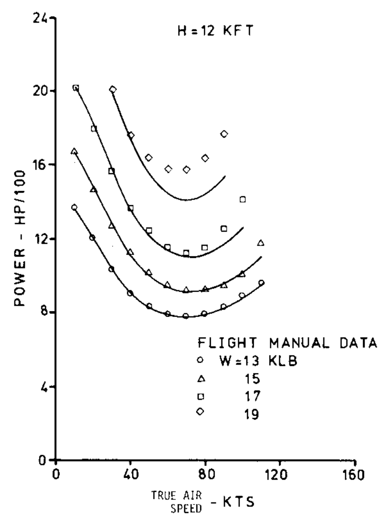

Fig. 2 Performance model fit at 12,000 ft altitude.

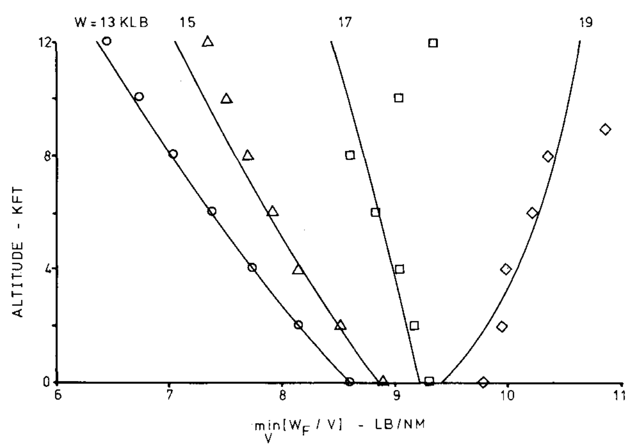

Fig. 3 Minimum fuel cruise cost: performance model and flight manual data.

which optimize some performance index. Typically we may be interested in minimum fuel or minimum operating cost, but in general, we assume the optimization problem can be formulated as minimization of an integral of the form

$$J=\int_0^{t_f} F\mathrm{d}t \tag{5}$$

The optimal trajectory is a determination of the altitude h and range x as functions of time as well as the supplementary variables such as speed, power setting, flight path angle, etc., such that performance index, Eq. (5), achieves a minimum value subject to a fixed range constraint and any additional constraints reflecting operating limits of the vehicle.

The problem as posed fits into the framework of a calculus of variations or "optimal control" problem. While necessary conditions for optimality are rather easy to specify (see, e.g., Bryson and Ho[9]), numerical computation of the minimizing trajectory can be a difficult, time-consuming task, involving iterative solutions of a nonlinear, two-point boundary value problem. This problem can be simplified if we assume at the outset an a priori stucture for the optimal trajectory. Using the methodology of Erzberger and Lee[4] we assume the trajectory can be split into three distinct segments as shown in Fig. 4. These are: 1) an ascent segment; 2) a constant altitude cruise segment; and 3) a descent segment. The cost integral can then be written as

$$J=\int_0^{t_{up}} (F)_{UP}\mathrm{d}t+\int_{t_{DST}}^{t_f} (F)_{DST}\mathrm{d}t+F_{CR}\Delta t_{CR} \tag{6}$$

To further simplify, we assume the energy per unit weight ("energy height")

$$h_e=h+V^2/2g \tag{7}$$

is monotonic on the ascent and descent portions of the trajectories and can be used as an independent variable. In the fixed-wing case used by Erzberger, the energy height satisfies

$$\mathrm{d}h_e/\mathrm{d}t=(P-P_R)/W \tag{8}$$

where P is the available power from the propulsive system and P_R is the power required for equilibrium (constant energy) flight. Also in Eq. (8), the acceleration normal to the flight path is considered small. To extend this concept to a rotary wing vehicle requires certain additional assumptions. First, on a helicopter, the rotor itself can serve as a significant energy storage device. If variable rotor speeds are to be considered then rotor energy should be an additional term in Eq. (8). For this study, a fixed-rotor speed which is consistent with the flight manual performance curves was used; hence this term is not present. Second, the performance model of the helicopter discussed in Section II differentiates changes in potential energy (h) by use of the climb and decent factors. This suggests that potential and kinetic energy may not be interchangeable on a trajectory, as is generally the case with a conventional aircraft. These complications can be suppressed

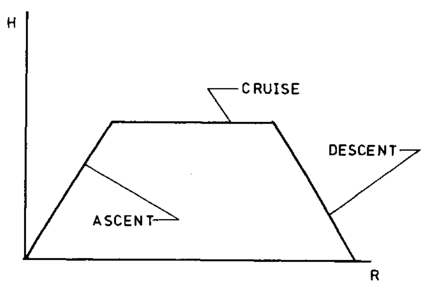

Fig. 4 Assumed structure of optimal trajectory.

by noting that since a helicopter is an inherently low-speed vehicle, the kinetic energy term and, in particular, the kinetic energy changes, are generally fairly small. Further, since the changes on the optimizing trajectories occur over a fairly substantial time period, it is reasonable then to assume that the kinetic energy term can be neglected completely. This also implies that the "energy state" to be used in this analysis is, simply, just the geometric altitude. Since the ascent and descent portions of the trajectory are disjoint, the energy relation, Eq. (8), can be modified for the helicopter by insertion of the respective climb or descent factor on the appropriate segment of the trajectory.

Converting to altitude as the independent variable, the cost function, Eq. (6), can be written as

$$J=\int_0^{h_{CR}} \left[\left(\frac{F}{\dot h}\right)_{UP}+\left(\frac{F}{-\dot h}\right)_{DST}\right]\mathrm{d}h+\left(\frac{F}{V+V_W}\right)_{CR}R_{CR} \tag{9}$$

where V_W is the wind (tailwind positive) and R_{CR} is the cruise range. In addition, the range must satisfy

$$R_F=\int_0^{t_{UP}} (V\cos\gamma+V_W)_{UP}\mathrm{d}t+R_{CR} + \int_{t_{DST}}^{t_F} (V\cos\gamma+V_W)_{DST}\mathrm{d}t$$

where γ is the flight path angle. Written in terms of h as the independent variable, this becomes

$$R_F=\int_0^{h_{CR}} \left[\left(\frac{V\cos\gamma+V_W}{\dot h}\right)_{UP} + \left(\frac{V\cos\gamma+V_W}{\dot h}\right)_{DST}+\right]\mathrm{d}h+R_{CR} \tag{10}$$

The integral constraint, Eq. (10), can be more easily treated by introducing a range variable $R(h)$ and rewriting as a differential equation constraint:

$$\frac{\mathrm{d}R}{\mathrm{d}h}=\left(\frac{V\cos\gamma+V_W}{\dot h}\right)_{UP}+\left(\frac{V\cos\gamma+V_W}{-\dot h}\right)_{DST} \tag{11}$$

$$R(0)=R_{CR} \qquad \text{(to be determined)} \tag{12}$$

$$R(h_{CR})=R_F \qquad \text{(given)} \tag{13}$$

The problem then becomes one of minimizing the integral of Eq. (9) while satisfying the differential equation constraint of Eq. (11) subject to boundary conditions, Eqs. (12) and (13). The control variables in the integral, Eq. (9), are the speed V and power available P on the climb and descent portions of the trajectory. Unlike the conventional energy state formulation, large flight path angles could have easily been incorporated into this analysis if desired. Since the minimizations are performed holding h fixed, γ can always be calculated explicitly as a function of V and P as $\gamma=\sin^{-1}(\dot h/V)$. In view of the other assumptions required in this analysis, this refinement was omitted, and the small angle assumption is used exclusively in the remainder of this Paper.

Using the Pontryagin Minimum Principle, this problem is solved by adjoining the differential equation constraint of Eq. (11) to Eq. (9), using a costate variable ψ. The resultant Hamiltonian can be distributed into climb and descent components as

$$H=H_{UP}+H_{DST} \tag{14}$$

where

$$H_{UP}=\left(\frac{F-(V+V_W)\psi}{\dot{h}}\right)_{UP} \tag{15}$$

$$H_{DST}=\left(\frac{F-(V+V_W)\psi}{-\dot{h}}\right)_{DST} \tag{16}$$

Additionally, we define the augmented function of end conditions

$$\psi_F=\left(\frac{F}{V+V_W}\right)_{CR}R_{CR}+\nu[R_f-R(h_{CR})] \tag{17}$$

H is a function of the variables V and P on both the ascent and descent segments of the trajectory. During cruise, we have the single control variable, V_{CR}. The wind V_W is a specified function of altitude on each of the three trajecjtory segments.

Application of the minimum principle leads to the following necessary conditions:

$$V^*_{UP},P^*_{UP}=\underset{V,P}{\operatorname{argmin}}\,(H_{UP}) \tag{18}$$

$$V^*_{DST},P^*_{DST}=\underset{V,P}{\operatorname{argmin}}\,(H_{DST}) \tag{19}$$

$$V_{CR}=\underset{V}{\operatorname{argmin}}\,(F/V+V_W) \tag{20}$$

Since H is not a function of R, the costate ψ is constant and is given by

$$\psi=\begin{cases}\min\limits_{V}\left(\dfrac{F}{V+V_W}\right)_{h=h_{CR}} & ;\quad R_{CR}\neq 0\\ \nu & ;\quad R_{CR}=0\end{cases} \tag{21}$$

If h_{CR} is a free variable, then the transversality condition at the optimal altitude is

$$H^*+\frac{\partial}{\partial h_{CR}}\left[\frac{F}{V+W}\right]R_{CR}=0 \tag{22}$$

For a further discussion of these conditions, see Erzberger.[4]

Properties of Optimal Controls

The minimization of H_{UP} and H_{DST} in Eq. (18) and (19) is a double minimization that must be performed at each altitude. For this study we look at functionals of the form

$$F=C_T+C_F W_f \tag{23}$$

where W_f is the fuel flow rate, and C_T and C_F are cost weighting factors for time and fuel, respectively. If $C_T=0$, we have the minimum fuel problem. If $C_F=0$, we have the minimum time problem. For the model under study, the fuel flow is a linear function of the power available, Eq. (4) in the form:

$$W_f=a+bP \tag{24}$$

Due to the linearity of Eqs. (23) and (24), the ascent and descent Hamiltonians are each bilinear functions of the power available. In the normal case, this then implies that the optimal power will be on a boundary of the admissible power region—a maximum on the ascent segment and a minimum on the descent segment of the optimal trajectory.

The optimal climb and descent velocities which minimize H_{UP} and H_{DST} are found from a numerical check of the respective Hamiltonians. The optimal speeds are sometimes in the interior of the admissible regions and sometimes will lie at a maximum speed boundary $V^*=V_{NE}$ (velocity not to be exceeded).

An Alternate Descent Limit

The use of a fixed minimum power limit may yield unacceptable descent rates in parts of the flight regime. Rather than specify $P_{\min}$, an alternate constraint which has been implemented is that of the descent rate

$$-\dot{h}_{DST}=\left(\frac{P_R-P}{W}\right)\leq\dot{h}_{\max}(\text{constant}) \tag{25}$$

In view of the previous discussions on the variation of H_{DST} with P, it can be seen that this inequality will be an equality on the optimum descent trajectory. This implies that on the descent $P^*_{DST}=P'_R-P_0$ where P_0 is a constant, depending on the descent rate. The descent Hamiltonian is of the form

$$H_{DST}=[C_T+C_F(P_R-P_0)-\psi V]/(\dot{h}_{\max}) \tag{26}$$

For the typical variation of P_R with V, Eq. (26) has a well defined minimum at $V=V^*_{DST}$. Generally H^*_{DST} is found to be negative. An important observation is that this minimizing speed is independent of the particular descent rate specified in Eq. (25).

Behavior of Optimal Solution as $h\to h_{CR}$

While the power and velocity which minimize the ascent and descent Hamiltonians can be found from the previous analysis, it is of interest to determine more precisely the behavior of the optimal solutions in the vicinity of the cruise altitude and to determine under what conditions a nonzero cruise segment will exist.

The previous results showed that for the normal problem, the optimum power will be a maximum on ascent and a minimum on descent. In this Section, we fix the power at these levels and examine the optimal speeds in the vicinity of h_{CR}. Consider specifically the ascent segment on which the optimal ascent speed is given by

$$V^*_{UP}=\underset{V_{UP}}{\operatorname{argmin}}\frac{F_{\max}-\psi V_{UP}}{(P_{\max}-P_R)} \tag{27}$$

where superfluous constants have been discarded from the Hamiltonian.

At the minimum the condition $(\mathrm{d}H_{UP}/\mathrm{d}V_{UP})$ leads to the relation

$$(P_{\max}-P_R)\psi-(F_{\max}-\psi V_{UP})\frac{\mathrm{d}P_R}{\mathrm{d}V_{UP}}=0 \tag{28}$$

For the assumed cost function and the linear fuel flow relation, we have

$$F_{\max}-F_R=C_F b(P_{\max}-P_R) \tag{29}$$

and

$$C_F b\frac{\mathrm{d}P_R}{\mathrm{d}V_{UP}}=\frac{\mathrm{d}F_R}{\mathrm{d}V_{UP}} \tag{30}$$

Using Eqs. (29) and (30) in Eq. (28), we have

$$\frac{\mathrm{d}F_R}{\mathrm{d}V_{UP}}=\left(\frac{F_{\max}-F_R}{F_{\max}-\psi V_{UP}}\right)\psi \tag{31}$$

The optimum V^*_{UP} is the solution to Eq. (31) at each $h<h_{CR}$. Since ψ minimizes F_{CR}/V_{CR}, we note that as $h\to h_{CR}$, then

$V_{UP}^* \rightarrow V_{CR}$ satisfies Eq. (31), since then

$$\frac{dF_R}{dV_{UP}} = \frac{dF_R}{dV_{CR}} = \psi$$

Our conclusion, then, is that the optimal ascent speed approaches the cruise speed as $h \rightarrow h_{CR}$. It is clear that the same result holds on any fixed power boundary; hence the optimal descent speed will similarly approach V_{CR}. By a slight modification of these arguments, the same conclusion is reached if the descent occurs on a fixed descent rate limit in Eq. (25).

While the optimal power is discontinuous at cruise, we see that as $h \rightarrow h_{CR}$ and $V \rightarrow V_{CR}$ the optimal Hamiltonian tends to become independent of P, leading to a nearly singular condition. Thus, while the limits of the minimizing values are discontinuous, the Hamiltonian itself is very insensitive to these variations at the point of discontinuity. For more detailed results, see Ref. 10.

Hamiltonian at Cruise

The optimal cruise altitude is determined from the transversality condition in Eq. (22), evaluated at h_{CR}.

It is of interest to examine the behavior of H^* to gain some insight into the nature of the cruise. Since $V_{UP} = V_{DST} = V_{CR}$ at $h = h_{CR}$, then $H^* = H_{UP}^* + H_{DST}^*$, which from Eqs. (15) and (16) becomes

$$H^* = \frac{F_{\max} - \psi V_{CR}}{k_{CLB}/W_{UP}(P_{\max} - P_{CR})} + \frac{F_{\min} - \psi V_{CR}}{k_{DST}/W_{DST}(P_{CR} - P_{\min})} \tag{32}$$

Using $\psi V_{CR} = F_{CR}$ and the linear fuel flow function, Eq. (32) can be simplified to

$$H^* = C_F b \left[\frac{W_{UP}}{k_{CLB}} - \frac{W_{DST}}{k_{DST}} \right] \tag{33}$$

Assume that for a short cruise segment $W_{UP} \simeq W_{DST} = W$. If we consider the case where $k_{CLB} = k_{DST} = 1$, then $H^* = 0$. From Eq. (22), $H^* = 0$ implies that the optimum cruise range $R_{CR} = 0$, except at the optimum cruise altitude h^*, where

$$h^* = \underset{h}{\operatorname{argmin}} [\psi] \tag{34}$$

At h^*, the condition of Eq. (22) gives $0 = 0$ and the optimum cruise range is determined from the total range constraint. Note that the two terms in H^* are H_{UP}^* and H_{DST}^*, implying that for this case these terms are equal in magnitude and opposite in sign. This is exactly the case found by Erzberger and Lee[4] for the fixed-wing case.

If the climb and descent factors are not equal, then

$$H^* = C_F b W \left[\frac{1}{k_{CLB}} - \frac{1}{k_{DST}} \right] \tag{35}$$

There exists a nozero cruise distance at all cruise altitudes given by

$$R_c = -H^* / \mathrm{d}/\mathrm{d}h_{CR} \left(\frac{F_{CR}}{V_{CR}} \right) \tag{36}$$

Equation (36) also confirms our earlier results that the cruise cost must be decreasing at the cruise altitude, since otherwise the cruise range would be negative. As $h_{CR} \rightarrow h^*$, the derivative $\mathrm{d}/\mathrm{d}h(F_{CR}/V_{CR}) \rightarrow 0$, while $H^* \neq 0$. Hence, the optimal cruise altitude for any fixed range trajectory is always *below* the altitude h^*.

IV. Numerical Results

Characteristics of Fuel Optimal Trajectories

A computer program was written which generates the optimal trajectories using the helicopter model and optimization algorithm developed in this paper. The program numerically integrates the climb and descent portions of the trajectory, simultaneously optimizing the ascent and descent Hamiltonians at each step. For the cruise segment, the cruise cost and speed are computed using a cruise weight computed as the average of the beginning and end weights on the cruise. Since these weights are not known initially, this process is done iteratively until the weights converge. Unless specifically mentioned otherwise, all results in this paper are for the case of minimum fuel cost in zero wind.

A typical set of trajectories is as shown in Fig. 5 for an initial weight of 14,000 lb. To generate these curves the cruise altitude was first specified; then the cruise range was calculated based on boundary conditions, Eq. (22), which must be satisfied at the optimal altitude and range. A descent rate constraint of 1800 ft/min (30 ft/s) was used on all trajectories. As a consequence of a nonunity climb factor ($k_{CLB} = 0.75$, $k_{DST} = 1.0$), all trajectories have a nonzero cruise range which, in fact, shows very little variation with altitude. The reason for the lack of variation is due primarily to the fact that the derivative $\mathrm{d}\psi/\mathrm{d}h$ is almost constant in this entire altitude range. Since the final Hamiltonian H^* is also relatively fixed, the subsequent cruise range computed from Eq. (22) exhibits very little variation with altitude.

For weights less than approximately 17,000 lb, the cruise cost is a monotonically decreasing function of altitude. Hence, for longer ranges, the optimal cruise altitude is increasingly higher. For the S-6IN the maximum allowable attitude is 12,000 ft. At this altitude, with an initial weight of 14,000 lb, the natural boundary condition on altitude yields a trajectory with total range of about 33 nm (see Fig. 5). For a trajectory with a specified range greater than this value, the minimum fuel trajectory cruises at the limit altitude of 12,000 ft. At this altitude, the cruise segment is determined not from the transversality condition, but from the total range condition

$$R_{CR} = R_F - (R_{UP} + R_{DST})$$

It is important to note that the mathematically optimum altitude is, quite significantly, a function of initial weight. The most important parameter in establishing the optimum altitude is the variation with altitude of the quasi-steady

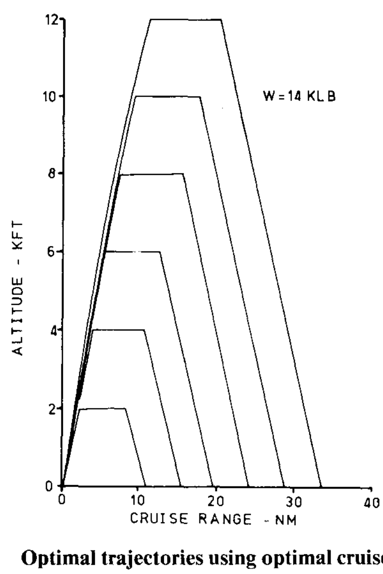

Fig. 5 Optimal trajectories using optimal cruise altitude.

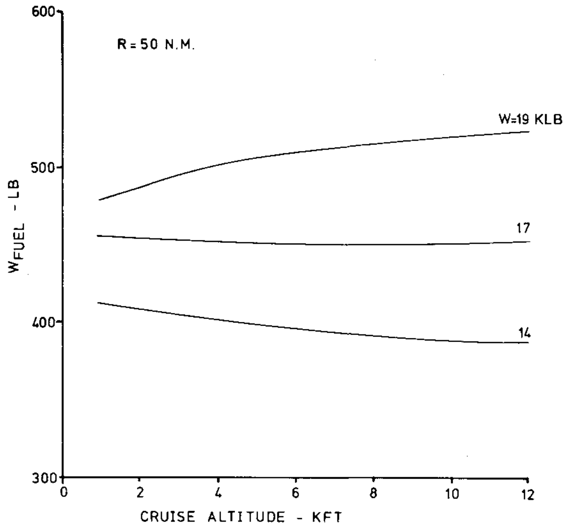

Fig. 6 Minimum fuel for 50-n.mi. trajectory for various cruise altitudes.

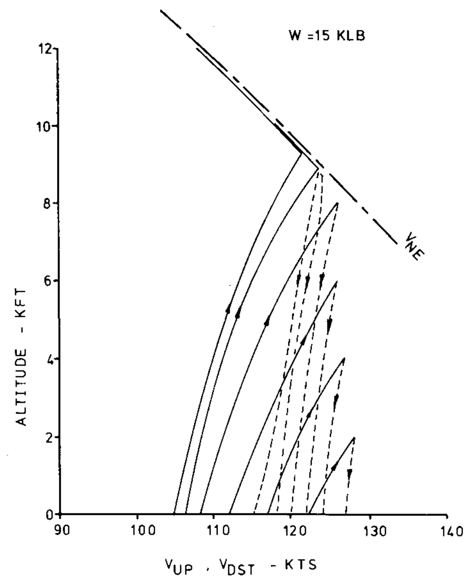

Fig. 8 Optimal ascent and descent speeds as functions of altitude for various cruise altitudes.

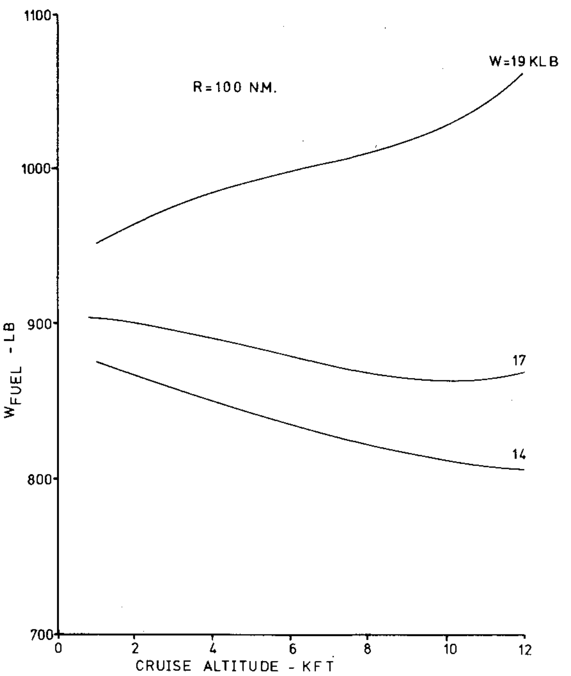

Fig. 7 Minimum fuel for 100-n.mi. trajectory as a function of cruise altitude.

"cruise cost" $\psi = \min_V (W_f/V)$. Using our variational approach, the optimum cruise altitude is at, or slightly below, the altitude for minimum ψ. Referring to Fig. 3, at low initial weights, the optimum cruise is at $h_{max} = 12{,}000$ ft. At high weights ($\approx 19{,}000$ lb) the optimum altitude is at $h_{min} \approx 0$.

Although the curves of cruise cost change rather smoothly with weight, the optimum altitude does change rather abruptly from h_{max} to h_{min} at a weight slightly greater than 17,000 lb. It should be pointed out that at the higher weights and higher altitudes, the simplified power model departs significantly from flight manual data. Thus, while at 17,000 lb the simplified model predicts improved cruise performance up to h_{max}, the flight manual data predicts deteriorating performance for altitudes greater than about 8,000 ft. Similarly, at the 19,000-lb weight, for altitudes above 8,000 ft, the simplified model predicts substantially better performance than the flight manual.

The significance of these differences can be more easily evaluated by observing the fuel variation on fixed range minimum fuel trajectories, where the cruise alttitude is constrained at a nonoptimal value as in Fig. 6. For the 14,000-lb vehicle, cruise at the optimal altitude of 12,000 ft represents about a 5% improvement over cruise at 1,000 ft. These results are felt to be reliable and represent a true fuel savings. For the heavy weight of 19,000 lb, the figure similarly predicts a substantial fuel savings of about 10% by choice of the lowest possible cruise altitude (here 1,000 ft). While the trend predicted in Fig. 6 is correct, the actual minimum fuel used is slightly in error, and the penalty for high cruise altitudes may be greater than that shown.

At the intermediate weight of 17,000 lb the total fuel consumed varies by only about 10 lb for all cruise altitudes from 1,000 to 12,000 ft. Thus, in this weight range, there is little to be gained by optimizing cruise altitude (of course velocity must still be optimized). In view of the model uncertainty at the high altitudes, conservative engineering judgment indicates that a cruise altitude below 8,000 ft is probably appropriate.

The identical curves for a 100 nm range mission are shown in Fig. 7. While the behavior is similar, we observe that there is slightly more variation in the 17,000-lb case. The reason for this is that the cruise fuel is based on the average cruise weight. Thus while the initial weights in Figs. 6 and 7 are the same, the "average cruise weight" for the 100 nm range is almost 200 lb less than in the former case. Note also that since the ψ's are different, the ascent and descent Hamiltonians are slightly different; therefore, the resulting climb and descent profiles are not exactly the same in the two cases. In general a smaller, more favorable cruise parameter ψ causes the ascent leg of the optimal trajectory to be steeper and to climb more quickly, making the climb segment shorter and the cruise segment longer.

Speed Variation on Optimal Trajectories

Optimal climb speeds on the minimum fuel trajectories tend to be much faster than the speeds for the fastest climb rate. At comparable altitudes, the speed in descent is generally faster than the speed in climb. The variation of the optimal speeds for one particular weight is shown in Fig. 8 for various cruise altitudes. For this and other results in this Section, the flight manual V_{NE} boundary (velocity not to be exceeded) was used as a velocity limit in the minimization of H. As shown in Fig.

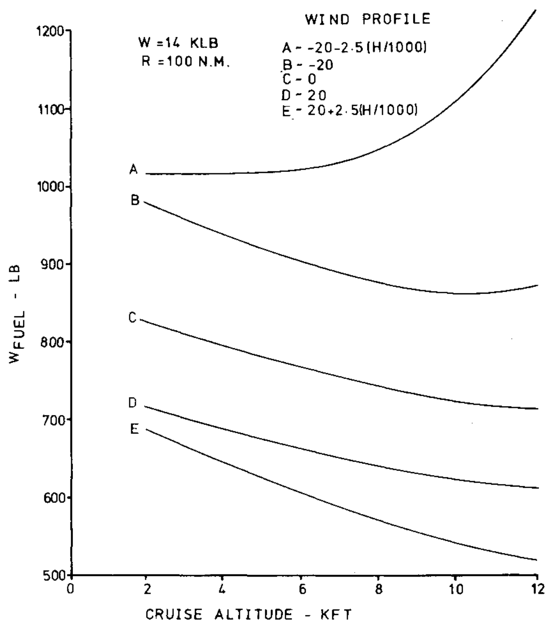

Fig. 9 Effect of wind on minimum fuel consumed on a fixed-range trajectory.

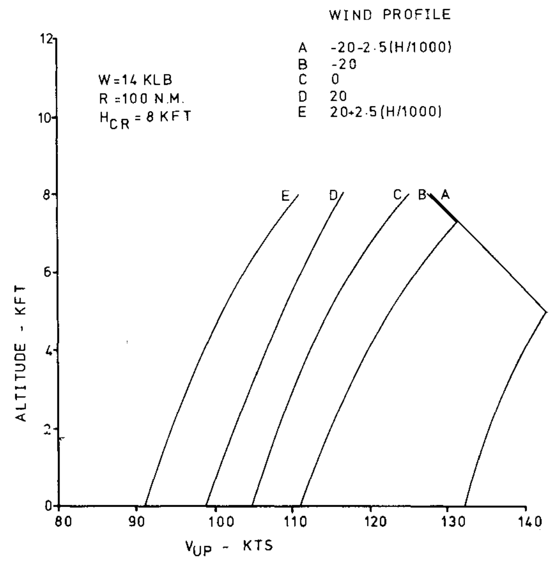

Fig. 10 Effect of wind on optimal ascent speed.

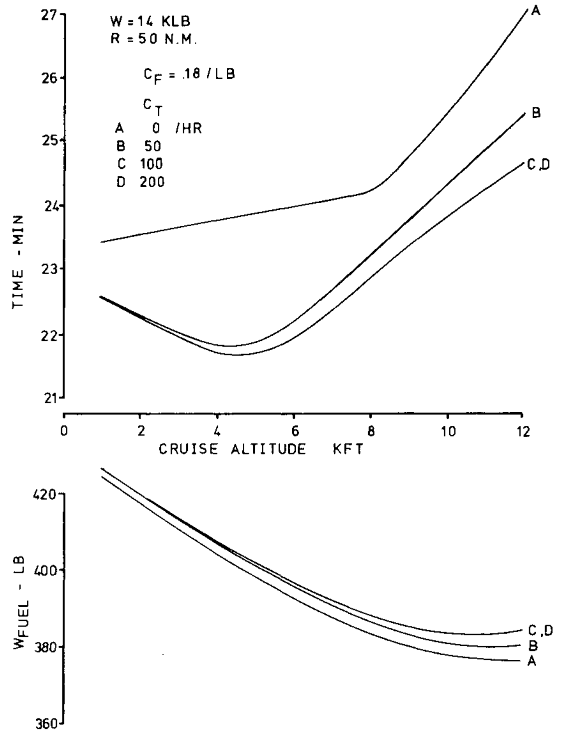

Fig. 11 Fuel and time on optimal trajectories for various time weights, C_T.

8, the V_{NE} constraint generally becomes active for flight at the higher altitudes. Numerical minimization of the Hamiltonians does confirm, in fact, that at the cruise altitude, the three speeds V_{UP}, V_{DST}, and V_{CR} all coalese to a single value. The advantage of trading off climb rate for forward speed can be seen in Table 1, which compares the fuel required for an optimal trajectory with the fuel required on a trajectory which uses a fastest climb on the ascent leg and an optimal cruise and descent. For the fastest climb rate, flight speeds are between 60-70 kt. true air speed.

While the advantage is slight, only about 2%, it should be pointed out that the fuel consumed in this case is dominated by the long cruise. In a shorter-range trajectory, it is anticipated that the relative advantage of the optimal profile will be increased.

Effect of Winds

The magnitude and direction of winds are known to have a substantial effect on the optimal performance of any aircraft. This effect is accentuated on a conventional helicopter because of the low flight speeds which make even a moderate wind fairly important. The effect of a tail wind tends to decrease the optimal air speed at all altitudes. The effect is reversed for a head wind. This is in accord with the generally accepted effect of wind on the quasi-steady performance quantities. Wind variation with altitude is also easily handled by the optimization algorithm, and again the altitude variation can have an important role in determining the optimal speed profile and the optimal altitude. Figures 9 and 10 show the fuel cost and the optimal climb speed for trajectories with various winds, including a linear wind where the wind velocity varies from 20 kt at $h=0$ to 50 kt at $h=12{,}000$ ft. While these winds are fairly substantial, they do represent, in fact, an environment not unusual for helicopters operating in offshore areas. The winds are added into the trajectory logic with little or no increase in complexity. The main difficulty with incorporating winds into an on-line procedure is the problem of determining the actual wind profile in the area of flight. If such knowledge can be obtained, the potential gain in fuel and range is substantial.

General Cost Optimization

The previous results have all applied to direct fuel minimization. The general cost function specified in this analysis allows a relative weighting of time and fuel. This can be used to minimize a "dollar" cost if an appropriate cost of time and fuel is specified. In addition, by varying the weight factor on time (C_T), the total flight time can be controlled. Determining an appropriate numerical value for C_F is done by simply inspecting the cost of fuel. For standard JP-4 at $1.16 per gallon at 6.5 lb/gal, this leads to $C_F = \$.18/\text{lb}$. C_T, the time weight factor, is a much more subtle number to try to generate. While direct operating costs are generally quoted as $500-$1,000 per h, the cost savings due to a decreased time are probably substantially less than this value.

Using $C_F = \$.18/\text{lb}$, the effects of C_T on the time of flight and the fuel required are plotted as functions of the cruise altitude in Fig. 11. Even when $C_T = 0$ (direct fuel minimization) the optimal speeds lie close to, or on, the V_{NE} boundary. The sudden slope discontinuity at $h_{CR} \simeq 8{,}000$ ft in

Table 1 Comparison of an optimal climb-cruise-descent profile with a fastest climb, optimal cruise-descent profile

Parameters	Optimal	Best R/C (ascent)
Range (n.mi)		
Up	11.9	5.5
Cruise	75.4	91.4
Descent	13.1	13.1
Total range	100.0	100.0
Fuel (lb)		
Up	145	117
Cruise	511	553
Descent	60	60
Total fuel	716	730

Fig. 11, on the $C_T=0$ curve, indicates the point where the optimal cruise speeds are being pushed back by the receding V_{NE} boundary, thereby increasing the time of flight. By making C_T positive, optimal speeds are increased. When C_T reaches \$100/h, the "optimal" speed is simply V_{NE} at all points on the trajectory, independent of the cruise altitude chosen. In this case, the minimum flight time occurs near a cruise altitude of 5,000 ft, where the maximum V_{NE} (for this weight) occurs. If the cruise altitude is fixed, increasing C_T beyond \$100/h cannot change the trajectory, since the velocity is already at a constraint boundary. The "cost" will certainly change, and, as shown in Fig. 11, the "optimum" cruise altitude will change, with larger values of C_T causing the optimal cruise altitude to drop to the minimum time altitude.

The appropriate values of C_T and C_F to use in a realistic environment must be determined by the helicopter operator. For this example with a fixed 50 nmi. range, the minimum fuel trajectory cruises at 12,000 ft and uses 375 lb fuel with a flight duration of 0.45 h or 27 min. For the minimum time trajectory, on the other hand, the cruise is at 5,000 ft, the total trip time is about 5 min shorter, and an extra 100 lb of fuel is consumed.

Recall that the fuel flow variation was of the form $W_f=a+bP$, where from (4), $a=400$ lb/h at sea level. Note that the effect of the zero power fuel flow a on the cost is exactly the same as a nonzero C_T; hence changes in the zero power fuel flow rate are reflected in thc cost curves and optimal trajectories for variable C_T.

V. Conclusion

The on-line determination of optimal flight paths for helicopters has been shown to be both useful and computationally feasible. The optimization procedure utilizes 1) an efficient, simple performance model for the helicopter, and 2) a simplified "climb-cruise-descent" trajectory optimization model.

For the model studied in this paper, the primary characteristics of the optimal trajectories can be summarized as follows:

1) The speeds on fuel optimal trajectories tend to be fairly high, that is, near the maximum speed of the vehicle. Speeds on the climb leg of the optimal trajectories tend to be significantly faster than the speed for greatest climb rate. As the length of the cruise segment increases, the optimal climb speed does, however, tend to approach that for greatest rate of climb.

2) All optimal helicopter trajectories have a nonzero cruise segment due to the nonunity climb factor used in the performance model. The cruise segment distance tends to be relatively constant at a small value, until the cruise altitude approaches the altitude for minimum cruise cost. In practice, the optimal altitudes are at sea level for the heavy weight case ($W \gtrsim 16{,}000$ lb).

3) The descent segment of the trajectory is always flown on a minimum power constraint of some type. While the fuel consumed in the descent segment is generally much less than on the remainder of the trajectory, the descent range is crucial in determining the length of the remaining trajectory segments.

4) Substantial decreases in time of flight can be achieved with only a small fuel penalty. Generally, the weighting of time on the optimal trajectory tends to force the cruise altitudes toward the altitude for greatest true air speed—about 5,000 ft for the vehicle in this study. Speeds in this case, of necessity, are faster than on the minimum fuel trajectory, and for the vehicle under consideration, are generally on a V_{NE} boundary.

Acknowledgment

This work was partially supported by NASA Ames Research Center under Grant NAG-2-175.

References

[1]Schmitz, F.H., "Optimal Take-Off Trajectories of a Heavily Loaded Helicopter," *Journal of Aircraft,* Vol. 8, Sept. 1971, pp. 717-723.

[3]Schmitz, F.H. and Vause, C.R., "A Simple, Near-Optimal Take-Off Control Policy for a Heavily Loaded Helicopter Operating from a Restricted Area," AIAA Paper No. 74-812, Aug. 1974.

[3]Olsen, J., "Helicopter Mission Optimization Study," NASA Contractor Rept. SER-50998, June 1978.

[4]Erzberger, H. and Lee, H., "Constrained Optimal Trajectories with Specified Range," *Journal of Guidance and Control,* Vol. 3, Jan.-Feb. 1980, pp. 78-85.

[5]Bramwell, A.R.S., *Helicopter Dynamics,* John Wiley & Sons, Inc., New York, 1976.

[6]Gessow, A. and Myers, G.C., *Aerodynamics of the Helicopter,* Ungar Publishing Co., New York, 1952, republished 1967.

[7]Flight Manual, Stratford, Conn., S-61N Helicopter, Sikorsky Aircraft Pub. #SA 4045-82.

[8]Keys, C.N., "Rotary Wing Aerodynamics, Vol. II—Performane Prediction of Helicopters," NASA Contractor Rept. 3083, Jan. 1979.

[9]Bryson, A.E. and Ho, Y.C., *Applied Optimal Control,* Blaisdell Publishing, Waltham, Mass., 1969.

[10]Slater, G.L. and Erzberger, H., "Optimal Short Range Trajectories for Helicopters," NASA TM 843-03, Dec. 1982.